In section 1-2, #36 the following model for a person's cholesterol level $C$ at time $t$ is considered:

$$\frac{dC}{dt} = k_1 (C_0 - C) + k_2 E,$$

where $t$ is time measured in days, $C(t)$ is the patient's cholesterol level at time $t$ in $mg$ decitl, $C_0$ is the patient's natural cholesterol level in $mg$ decitl, $k_1$ is a parameter for cholesterol production of the patient's body, $k_2$ is a parameter for the absorption of cholesterol from food, and $E$ is the daily rate at which cholesterol is eaten (via food) in $mg$ day.

(a,b) Suppose that $C_0 = 200$, $k_1 = k_2 = 0.1$, $E = 400$, $C(0) = 150$. What will the person's cholesterol level be after 2, 5 days on this diet?

To solve the differential equation, we observe that the variables can be separated, viewing the r.h.s as a function $C$ and the function of to be $g(t) = 1$. Using the technique, we will solve the equation

$$\int_{k_1 (C_0 - C) + k_2 E}^1 dC = \int^1 dt$$

$$\int_{-k_1 C - k_1 C_0 + k_2 E}^{t + C_1} dC = t + C_1,$$

and with the substitution $u = -k_1 C - k_1 C_0 + k_2 E$, $du = -k_1 dC$, this becomes

$$- \frac{1}{k_1} \int \frac{1}{u} du = t + C_1$$

$$- \frac{1}{k_1} ln | - k_1 C - k_1 C_0 + k_2 E| = t + C_1$$

$$ln | - k_1 C - k_1 C_0 + k_2 E| = - k_1 t + C_2$$

$$| - k_1 C - k_1 C_0 + k_2 E| = e^{-k_1 t + C_2} = C_3 e^{-k_1 t}$$

$$- k_1 C - k_1 C_0 + k_2 E = C_4 e^{-k_1 t}$$

$$C = - \frac{C_4}{k_1} e^{-k_1 t} + C_0 + \frac{k_2}{k_1} E$$

so that

$$C(t) = Ce^{-k_1 t} + C_0 + \frac{k_2}{k_1} E \quad (1)$$

for an arbitrary constant $C$, to be determined from initial conditions. Given $C_0 = 200$, $k_1 = k_2 = 0.1$, $E = 400$, and $C(0) = 150$ we obtain that $C = - 450$, and thus

$$C(t) = - 450 e^{-0.1t} + 600.$$

From here then we obtain that $C(2) \approx 232$ and that $C(5) \approx 328.$
(c) With the initial conditions as above, what will the person's cholesterol level be after a long time on this diet?

Clearly, as \( t \to \infty \), \( C(t) \to 600 \). \( \ldots \text{since } e^{-x} \to 0 \text{ as } x \to \infty \}

(d) Suppose that, after a long time on the high cholesterol diet described above, the person goes on a very low cholesterol diet, so \( E \) changes to \( E = 100 \). What will the person's cholesterol level be after 1 day on the new diet, after 5 days on the new diet, and after a very long time on the new diet?

When the diet is changed, which we will consider as \( t = 0 \), the cholesterol level is \( C(0) = 600 \) mg. The new value for \( E \) is 100, and we saw above that the general solution is

\[
C(t) = Ce^{-kt} + C_0 + \frac{k_2}{k_1}E. \tag{1}
\]

Since \( C_0 \) is still 200, we obtain upon inserting the parameter values that

\[
C(t) = Ce^{-0.1t} + 300.
\]

The initial condition \( C(0) = 600 \) tells us that \( C = 300 \), and our particular solution is

\[
C(t) = 300e^{-0.1t} + 300.
\]

Consequently we will have \( C(1) \approx 571 \), \( C(5) \approx 482 \), and \( C(t) \to 300 \) as \( t \to \infty \).

(e) Suppose the person stays on the high cholesterol diet, but takes drugs that block some of the uptake of cholesterol from food, so \( k_2 = 0.075 \). Starting with the cholesterol level from part (c), what will the person's cholesterol level be after 1 day, after 5 days, and after a very long time?

Since the patient remains on the high cholesterol diet, \( E = 400 \), and the drug causes \( k_2 = 0.075 \). With these changes we obtain from our general solution

\[
C(t) = Ce^{-kt} + C_0 + \frac{k_2}{k_1}E. \tag{1}
\]

our particular solution

\[
C(t) = Ce^{-0.1t} + (200 + \frac{0.075}{0.1}400) = Ce^{-0.1t} + 500,
\]

and the initial condition \( C(0) = 600 \) leads to

\[
C(t) = 100e^{-0.1t} + 500.
\]

Therefore, \( C(1) \approx 590 \), \( C(5) \approx 560 \), and \( C(t) \to 500 \) as \( t \to \infty \).